Roll No. _______ **Code No. Q-4311**

SECOND SEMESTER EXAMINATION 2021-22

M.Sc. - MATHEMATICS

Paper - I

Advanced Abstract Algebra-II

Time : 3.00 Hrs. **Max. Marks : 80**

Total No. of Printed Page : 03 **Mini. Marks : 29**

Note: Question paper is divided into three sections. Attempt question of all three section as per direction. Distribution of Marks is given in each section.

Section - 'A'

Very short type question (in few words). **6x2=12**

Q.1 Attempt any six question from the following questions :

(i) Define finitely congenerated Module.

(ii) Show that Infinite Cyclic groups are not artinian.

(iii) Let A be an mxn matrix over PID. Then show that rank A=column rank A.

(iv) Define Torsion Element.

(v) Define Noetherian Module.

(vi) Show that Z is a noetherian ring but not an artinian - ring.

(vii) Define Semi - Simple ring.

(viii) Define Row Rank and Column Rank of a Matrix.

(ix) Define Principal Right Ideal.

(x) Give only statement of Third isomorphism theorem.

Section - 'B'

Short answer question (In 200 words) **4x5=20**

Q.2 Attempt any four question from the following questions :

(i) Every homomorphic image of an artinian module is artinian.

(ii) Let N be a nil ideal in a noetherian ring R Then N is nilpotent.

(iii) Obtain the Smith normal form and rank for the matrix with integral entries.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}$$

(iv) State and proof "First isomorphsm theorem".

(v) Find the abelian group generated by (x_1, x_2, x_3) subject to

$$5x_1 + 9x_2 + 5x_3 = 0$$
$$2x_1 + 4x_2 + 3x_3 = 0$$
$$x_1 + x_2 - 3x_3 = 0$$

(vi) Show that the abelian group generated by x_1 and x_2 subject to $2x_1 = 0,\ 3x_2 = 0$ is isomorphic to z/(6).

(vii) Show that $Z / (P_1P_2)$ is completely reducible Z - module, where P_1 and P_2 are distinct primes.

Section - 'C'

Long answer/Essay type question. **4x12=48**

Q.3 Attempt any four question from the following questions :

(i) Prove that the intersection of all prime ideals in a noetherian ring is nilpotent.

(ii) Let $R = \left\{ \begin{pmatrix} x & y \\ o & o \end{pmatrix} | \ x, y \varepsilon \theta \right\}$ Show that R is a left artinian ring (without unity) but not right artinian.

(iii) Find the invariant factors of the matrix.

$$\begin{bmatrix} -x & 4 & -2 \\ -3 & 8-x & 3 \\ .4 & -8 & -2-x \end{bmatrix}$$ over the

ring Q [x]. Also find the rank.

(iv) Let M be a finitely generated module over a principal ideal domain R.

Then M = F $\oplus$ Tor M.

Where (i) $F \simeq R^s$ for some non-negative integer $\wp$ and

(ii) Tor $M \simeq R/Ra, \oplus ..... \oplus R/R\ \ ar$ where

ai are non - zero non-unit elements in R such that $a_1 | a_2 | ..... | a_r$

(v) If R is a noetherian ring then show that ab=1, a, bε R if and only if ba = 1.

(vi) Prove that a Boolean noetherian ring is finite and is a finite direct product of fields with two elements.

(vii) Find the invariant factors of the matrix

$$\begin{bmatrix} x+1 & 2 & -6 \\ 1 & x & -3 \\ 1 & 1 & x-4 \end{bmatrix}$$

over the ring $Q[n]$. Also find the rank.

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